Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

1. $f(x)=3x+8, g(x)=\frac{x-8}{3}$

SOLUTION:

$$\begin{aligned}[f\circ g](x)&=f(g(x))\\&=f\left(\frac{x-8}{3}\right)\\&=3\left(\frac{x-8}{3}\right)+8\\&=x-8+8\\&=x\end{aligned}$$

$$\begin{aligned}[g\circ f](x)&=g(f(x))\\&=g(3x+8)\\&=\frac{(3x+8)-8}{3}\\&=\frac{3x}{3}\\&=x\end{aligned}$$

Since $[f\circ g](x)=[g\circ f](x)=x$, they are inverse functions.

2. $f(x)=\frac{1}{3}x+5, g(x)=3x-15$

SOLUTION:

$$\begin{aligned}[f\circ g](x)&=f(g(x))\\&=f(3x-15)\\&=\frac{1}{3}(3x-15)+5\\&=x-5+5\\&=x\end{aligned}$$

$$\begin{aligned}[g\circ f](x)&=g(f(x))\\&=g\left(\frac{1}{3}x+5\right)\\&=3\left(\frac{1}{3}x+5\right)-15\\&=x+15-15\\&=x\end{aligned}$$

Since $[f\circ g](x)=[g\circ f](x)=x$, they are inverse functions.

3. $f(x)=x+7, g(x)=x-7$

SOLUTION:

$$\begin{aligned}[f\circ g](x)&=f(g(x))\\&=f(x-7)\\&=(x-7)+7\\&=x-7+7\\&=x\end{aligned}$$

$$\begin{aligned}[g\circ f](x)&=g(f(x))\\&=g(x+7)\\&=(x+7)-7\\&=x+7-7\\&=x\end{aligned}$$

Since $[f\circ g](x)=[g\circ f](x)=x$, they are inverse functions.

4. $g(x) = 3x - 2, f(x) = \frac{x-2}{3}$

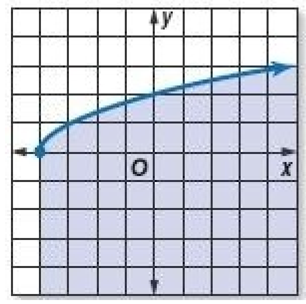
SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f(g(x)) \\ &= f(3x-2) \\ &= \frac{3x-2-2}{3} \\ &= x - \frac{4}{3}\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g(f(x)) \\ &= g\left(\frac{x-2}{3}\right) \\ &= 3\left(\frac{x-2}{3}\right) - 2 \\ &= x-2-2 \\ &= x-4\end{aligned}$$

Since $[f \circ g](x) \neq [g \circ f](x)$, they are *not* inverse functions.

5. **MULTIPLE CHOICE** Which inequality represents the graph below?



A $y \geq \sqrt{x+4}$

B $y \leq \sqrt{x+4}$

C $y \geq \sqrt{x-4}$

D $y \leq \sqrt{x-4}$

SOLUTION:

The graph represents the inequality $y \leq \sqrt{x+4}$.

Option B is the correct answer.

If $f(x) = 3x + 2$ and $g(x) = x^2 - 2x + 1$, find each function.

6. $(f+g)(x)$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= 3x+2+x^2-2x+1 \\ &= x^2+x+3\end{aligned}$$

7. $(f \cdot g)(x)$

SOLUTION:

$$\begin{aligned}(f \cdot g)(x) &= (3x+2)(x^2-2x+1)\\ &= 3x^3-6x^2+3x+2x^2-4x+2\\ &= 3x^3-4x^2-x+2\end{aligned}$$

8. $(f-g)(x)$

SOLUTION:

$$\begin{aligned}(f-g)(x) &= (3x+2)-(x^2-2x+1)\\ &= 3x+2-x^2+2x-1\\ &= -x^2+5x+1\end{aligned}$$

9. $\left(\frac{f}{g}\right)(x)$

SOLUTION:

$$\left(\frac{f}{g}\right)(x) = \frac{3x+2}{x^2-2x+1} \text{ for } x \neq 1$$

Solve each equation.

10. $\sqrt{a+12} = \sqrt{5a-4}$

SOLUTION:

$$\begin{aligned}\sqrt{a+12} &= \sqrt{5a-4}\\ \left(\sqrt{a+12}\right)^2 &= \left(\sqrt{5a-4}\right)^2\\ a+12 &= 5a-4\\ 4a &= 16\\ a &= 4\end{aligned}$$

11. $\sqrt{3x} = \sqrt{x-2}$

SOLUTION:

$$\begin{aligned}\sqrt{3x} &= \sqrt{x-2}\\ \left(\sqrt{3x}\right)^2 &= \left(\sqrt{x-2}\right)^2\\ 3x &= x-2\\ 2x &= -2\\ x &= -1\end{aligned}$$

For $x = -1$, the radical $\sqrt{3x}$ become undefined.

Therefore, there is no solution.

12. $4\left(\sqrt[4]{3x+1}\right)-8=0$

SOLUTION:

$$\begin{aligned}4\left(\sqrt[4]{3x+1}\right)-8 &= 0\\ 4\left(\sqrt[4]{3x+1}\right) &= 8\\ \sqrt[4]{3x+1} &= 2\\ \left(\sqrt[4]{3x+1}\right)^4 &= 2^4\\ 3x+1 &= 16\\ 3x &= 15\\ x &= 5\end{aligned}$$

13. $\sqrt[3]{5m+6}+15=21$

SOLUTION:

$$\begin{aligned}\sqrt[3]{5m+6}+15 &= 21\\ \sqrt[3]{5m+6} &= 6\\ \left(\sqrt[3]{5m+6}\right)^3 &= 6^3\\ 5m+6 &= 216\\ 5m &= 210\\ m &= 42\end{aligned}$$

14. $\sqrt{3x+21}=\sqrt{5x+27}$

SOLUTION:

$$\sqrt{3x+21}=\sqrt{5x+27}$$
$$\left(\sqrt{3x+21}\right)^2=\left(\sqrt{5x+27}\right)^2$$
$$3x+21=5x+27$$
$$2x=-6$$
$$x=-3$$

15. $1+\sqrt{x+11}=\sqrt{2x+15}$

SOLUTION:

$$1+\sqrt{x+11}=\sqrt{2x+15}$$
$$\left(1+\sqrt{x+11}\right)^2=\left(\sqrt{2x+15}\right)^2$$
$$1+x+11+2\sqrt{x+11}=2x+15$$
$$2\sqrt{x+11}=x+3$$
$$\left(2\sqrt{x+11}\right)^2=(x+3)^2$$
$$4(x+11)=x^2+6x+9$$
$$x^2+2x-35=0$$
$$(x+7)(x-5)=0$$

By Zero Product Property:

$x+7=0$	or	$x-5=0$
$x=-7$	or	$x=5$

Check:

$$1+\sqrt{-7+11}\stackrel{?}{=}\sqrt{2(-7)+15}$$
$$1+\sqrt{4}\stackrel{?}{=}\sqrt{-14+15}$$
$$1+2\stackrel{?}{=}1$$
$$3\neq 1 \times$$
$$1+\sqrt{5+11}\stackrel{?}{=}\sqrt{2(5)+15}$$
$$1+\sqrt{16}\stackrel{?}{=}\sqrt{10+15}$$
$$1+4\stackrel{?}{=}5$$
$$5=5 \checkmark$$

The solution is 5.

16. $\sqrt{x-5}=\sqrt{2x-4}$

SOLUTION:

$$\begin{aligned}\sqrt{x-5}&=\sqrt{2x-4}\\\left(\sqrt{x-5}\right)^2&=\left(\sqrt{2x-4}\right)^2\\x-5&=2x-4\\x&=-1\end{aligned}$$

For $x = -1$, the function $\sqrt{x-5}$ is undefined. Therefore, there is no solution.

17. $\sqrt{x-6}-\sqrt{x}=3$

SOLUTION:

$$\begin{aligned}\sqrt{x-6}-\sqrt{x}&=3\\\left(\sqrt{x-6}-\sqrt{x}\right)^2&=3^2\\x-6+x-2\sqrt{x^2-6x}&=9\\2\sqrt{x^2-6x}&=2x-15\\\left(2\sqrt{x^2-6x}\right)&=(2x-15)^2\\4x^2-24x&=4x^2-60x+225\\36x&=225\\x&=\frac{25}{4}\end{aligned}$$

Check:

$$\begin{aligned}\sqrt{\frac{25}{4}-6}-\sqrt{\frac{25}{4}}&\stackrel{?}{=}3\\\sqrt{\frac{25-24}{4}}-\sqrt{\frac{25}{4}}&\stackrel{?}{=}3\\\sqrt{\frac{1}{4}}-\sqrt{\frac{25}{4}}&\stackrel{?}{=}3\\\frac{1}{2}-\frac{5}{2}&\stackrel{?}{=}3\\\frac{1-5}{2}&\stackrel{?}{=}3\\\frac{-4}{2}&\stackrel{?}{=}3\\-2&\neq 3\end{aligned}$$

There exists no solution for the equation.

18. **MULTIPLE CHOICE** Which expression is equivalent to $125^{-\frac{1}{3}}$?

F –5

G $-\frac{1}{5}$

H $\frac{1}{5}$

J 5

SOLUTION:

$$\begin{aligned}125^{-\frac{1}{3}} &= \frac{1}{125^{\frac{1}{3}}}\\ &= \frac{1}{\left(5^3\right)^{\frac{1}{3}}}\\ &= \frac{1}{5}\end{aligned}$$

Option H is the correct answer.

Simplify.

19. $\left(2+\sqrt{5}\right)\left(6-3\sqrt{5}\right)$

SOLUTION:

$$\begin{aligned}\left(2+\sqrt{5}\right)\left(6-3\sqrt{5}\right) &= 12-6\sqrt{5}+6\sqrt{5}-3\sqrt{5^2}\\ &= 12-15\\ &= -3\end{aligned}$$

20. $\left(3-2\sqrt{2}\right)\left(-7+\sqrt{2}\right)$

SOLUTION:

$$\begin{aligned}\left(3-2\sqrt{2}\right)\left(-7+\sqrt{2}\right) &= -21+3\sqrt{2}+14\sqrt{2}-2\sqrt{2^2}\\ &= -21+17\sqrt{2}-4\\ &= -25+17\sqrt{2}\end{aligned}$$

21. $\frac{12}{2-\sqrt{3}}$

SOLUTION:

$$\begin{aligned}\frac{12}{2-\sqrt{3}} &= \frac{12}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}\\ &= \frac{12\left(2+\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}\\ &= \frac{24+12\sqrt{3}}{4-3}\\ &= 24+12\sqrt{3}\end{aligned}$$

22. $\frac{m^{\frac{1}{2}}-1}{2m^{\frac{1}{2}}+1}$

SOLUTION:

$$\begin{aligned}\frac{m^{\frac{1}{2}}-1}{2m^{\frac{1}{2}}+1} &= \frac{\sqrt{m}-1}{2\sqrt{m}+1}\\ &= \frac{\sqrt{m}-1}{2\sqrt{m}+1}\cdot\frac{2\sqrt{m}-1}{2\sqrt{m}-1}\\ &= \frac{2\sqrt{m^2}-\sqrt{m}-2\sqrt{m}+1}{\left(2\sqrt{m}\right)^2-1^2}\\ &= \frac{2m-3\sqrt{m}+1}{4m-1}\\ &= \frac{2m-3m^{\frac{1}{2}}+1}{4m-1}\end{aligned}$$

23. $4\sqrt{3}-8\sqrt{48}$

SOLUTION:

$$\begin{aligned}4\sqrt{3}-8\sqrt{48} &= 4\sqrt{3}-8\sqrt{16\cdot 3}\\ &= 4\sqrt{3}-8\sqrt{4^2\cdot 3}\\ &= 4\sqrt{3}-32\sqrt{3}\\ &= -28\sqrt{3}\end{aligned}$$

24. $5^{\frac{2}{3}}\cdot 5^{\frac{1}{2}}\cdot 5^{\frac{5}{6}}$

SOLUTION:

$$\begin{aligned}5^{\frac{2}{3}}\cdot 5^{\frac{1}{2}}\cdot 5^{\frac{5}{6}} &= 5^{\frac{2}{3}+\frac{1}{2}+\frac{5}{6}}\\ &= 5^{\frac{12}{6}}\\ &= 5^2\\ &= 25\end{aligned}$$

25. $\sqrt[6]{729a^9b^{24}}$

SOLUTION:

$$\begin{aligned}\sqrt[6]{729a^9b^{24}} &= \left(729a^9b^{24}\right)^{\frac{1}{6}}\\ &= \left(3^6\cdot a^6\cdot a^3\cdot\left(b^4\right)^6\right)^{\frac{1}{6}}\\ &= 3aa^{\frac{1}{2}}b^4\\ &= 3ab^4\sqrt{a}\end{aligned}$$

26. $\sqrt[5]{32x^{15}y^{10}}$

SOLUTION:

$$\begin{aligned}\sqrt[5]{32x^{15}y^{10}} &= \left(32x^{15}y^{10}\right)^{\frac{1}{5}}\\ &= \left(2^5\left(x^3\right)^5\left(y^2\right)^5\right)^{\frac{1}{5}}\\ &= 2x^3y^2\end{aligned}$$

27. $w^{-\frac{4}{5}}$

SOLUTION:

$$\begin{aligned}w^{-\frac{4}{5}} &= \frac{1}{w^{\frac{4}{5}}}\\ &= \frac{1}{\sqrt[5]{w^4}}\\ &= \frac{1}{\sqrt[5]{w^4}}\cdot\frac{\sqrt[5]{w}}{\sqrt[5]{w}}\\ &= \frac{\sqrt[5]{w}}{w}\\ &= \frac{w^{\frac{1}{5}}}{w}\end{aligned}$$

28. $\dfrac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}}$

SOLUTION:

$$\frac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}} = \frac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}} \cdot \frac{r^{\frac{5}{6}}}{r^{\frac{5}{6}}} = \frac{r^{\frac{9}{6}}}{r^{\frac{6}{6}}} = \frac{r \cdot r^{\frac{1}{2}}}{r} = r^{\frac{1}{2}}$$

29. $\dfrac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}} \cdot a^{-\frac{1}{4}}}$

SOLUTION:

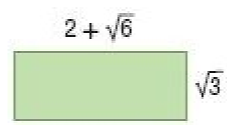
$$\frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}} \cdot a^{-\frac{1}{4}}} = \frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{12}}} = \frac{a^{-\frac{1}{2}}}{6a^{\frac{1}{12}}} \cdot \frac{a^{\frac{11}{12}}}{a^{\frac{11}{12}}} = \frac{a^{\frac{5}{12}}}{6a}$$

30. $\dfrac{y^{\frac{3}{2}}}{y^{\frac{1}{2}} + 2}$

SOLUTION:

$$\frac{y^{\frac{3}{2}}}{y^{\frac{1}{2}} + 2} = \frac{y^{\frac{3}{2}}}{y^{\frac{1}{2}} + 2} \cdot \frac{y^{\frac{1}{2}} - 2}{y^{\frac{1}{2}} - 2} = \frac{y^2 - 2y^{\frac{3}{2}}}{\left(y^{\frac{1}{2}}\right)^2 - 2^2} = \frac{y^2 - 2y^{\frac{3}{2}}}{y - 4}$$

31. **MULTIPLE CHOICE** What is the area of the rectangle?



A $2\sqrt{3} + 3\sqrt{2}$ units^2

B $4 + 2\sqrt{6} + 2\sqrt{3}$ units^2

C $2\sqrt{3} + \sqrt{6}$ units^2

D $2\sqrt{3} + 3$ units^2

SOLUTION:

$$\text{Area} = \left(2 + \sqrt{6}\right)\sqrt{3} = 2\sqrt{3} + \sqrt{18} = 2\sqrt{3} + 3\sqrt{2} \text{ unit}^2$$

Option A is the correct answer.

Solve each inequality.

32. $\sqrt{4x-3}<5$

SOLUTION:
Since the radicand of a square root must be greater than or equal to zero, first solve $4x-3\geq 0$.

$$\begin{aligned} 4x-3&\geq 0\\ 4x&\geq 3\\ x&\geq \frac{3}{4}\end{aligned}$$

Solve $\sqrt{4x-3}<5$.

$$\begin{aligned} \sqrt{4x-3}&<5\\ \left(\sqrt{4x-3}\right)^2&<5^2\\ 4x-3&<25\\ 4x&<28\\ x&<7\end{aligned}$$

The solution region is $\frac{3}{4}\leq x<7$.

33. $-2+\sqrt{3m-1}<4$

SOLUTION:
Since the radicand of a square root must be greater than or equal to zero, first solve $3m-1\geq 0$.[

$$\begin{aligned} 3m-1&\geq 0\\ 3m&\geq 1\\ m&\geq \frac{1}{3}\end{aligned}$$

Solve $-2+\sqrt{3m-1}<4$.

$$\begin{aligned} -2+\sqrt{3m-1}&<4\\ \sqrt{3m-1}&<6\\ \left(\sqrt{3m-1}\right)^2&<6^2\\ 3m-1&<36\\ 3m&<37\\ m&<\frac{37}{3}\end{aligned}$$

The solution region is $\frac{1}{3}\leq m<\frac{37}{3}$.

34. $2+\sqrt{4x-4}\leq 6$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $4x-4\geq 0$.

$$\begin{aligned}4x-4&\geq 0\\4x&\geq 4\\x&\geq 1\end{aligned}$$

Solve $2+\sqrt{4x-4}\leq 6$.

$$\begin{aligned}2+\sqrt{4x-4}&\leq 6\\\sqrt{4x-4}&\leq 4\\\left(\sqrt{4x-4}\right)^2&\leq 4^2\\4x-4&\leq 16\\4x&\leq 20\\x&\leq 5\end{aligned}$$

The solution region is $1\leq x\leq 5$.

35. $\sqrt{2x+3}-4\leq 5$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $2x+3\geq 0$.

$$\begin{aligned}2x+3&\geq 0\\2x&\geq -3\\x&-\frac{3}{2}\end{aligned}$$

Solve $\sqrt{2x+3}-4\leq 5$.

$$\begin{aligned}\sqrt{2x+3}-4&\leq 5\\\sqrt{2x+3}&\leq 9\\\left(\sqrt{2x+3}\right)^2&\leq 9^2\\2x+3&\leq 81\\2x&\leq 78\\x&\leq 39\end{aligned}$$

The solution region is $-\frac{3}{2}\leq x\leq 39$.

36. $\sqrt{b+12}-\sqrt{b}>2$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $b+12\geq 0$ and $b\geq 0$.

$$\begin{aligned} b+12 &\geq 0 \quad \text{or} \quad b\geq 0 \\ b &\geq -12 \quad \text{or} \quad b\geq 0 \end{aligned}$$

Solve $\sqrt{b+12}-\sqrt{b}>2$.

$$\begin{aligned} \sqrt{b+12}-\sqrt{b} &> 2 \\ \left(\sqrt{b+12}-\sqrt{b}\right)^2 &> 2^2 \\ b+12+b-2\sqrt{b^2+12b} &> 4 \\ \sqrt{b^2+12b} &< b+4 \\ \left(\sqrt{b^2+12b}\right)^2 &< (b+4)^2 \\ b^2+12b &< b^2+16+8b \\ 4b &< 16 \\ b &< 4 \end{aligned}$$

The solution region is $0\leq b<4$.

37. $\sqrt{y-7}+5\geq 10$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $y-7\geq 0$.

$$\begin{aligned} y-7 &\geq 0 \\ y &\geq 7 \end{aligned}$$

Solve $\sqrt{y-7}+5\geq 10$.

$$\begin{aligned} \sqrt{y-7}+5 &\geq 10 \\ \sqrt{y-7} &\geq 5 \\ \left(\sqrt{y-7}\right)^2 &\geq 5^2 \\ y-7 &\geq 25 \\ y &\geq 32 \end{aligned}$$

The solution region is $y\geq 32$.

38. $\sqrt{a-5}-\sqrt{a+7}\le 4$

SOLUTION:

Since the radicand of a square root must be greater than or equal to zero, first solve $a-5\ge 0$ and $a+7\ge 0$.

$$\begin{aligned} a-5&\ge 0 \quad \text{or} \quad a+7\ge 0\\ a&\ge 5 \quad \text{or} \quad a\ge -7 \end{aligned}$$

The common region between the inequalities $a\ge 5$ or $a\ge -7$ is $a\ge 5$.

Solve $\sqrt{a-5}-\sqrt{a+7}\le 4$.

$$\begin{aligned} \sqrt{a-5}-\sqrt{a+7}&\le 4\\ \left(\sqrt{a-5}-\sqrt{a+7}\right)^2&\le 4^2\\ a-5+a+7-2\sqrt{a^2+2a-35}&\le 16\\ 2a+2-2\sqrt{a^2+2a-35}&\le 16\\ a+1-\sqrt{a^2+2a-35}&\le 8\\ -\sqrt{a^2+2a-35}&\le -a+7\\ \sqrt{a^2+2a-35}&\ge a-7\\ \left(\sqrt{a^2+2a-35}\right)^2&\ge (a-7)^2\\ a^2+2a-35&\ge a^2+49-14a\\ 16a&\ge 84\\ a&\ge 5.25 \end{aligned}$$

The solution region is $a\ge 5.25$.

39. $\sqrt{c+5}+\sqrt{c+10}>2$

SOLUTION:

$$\begin{aligned} c+5&>0\\ c&>-5 \end{aligned}$$

$$\begin{aligned} c+10&>0\\ c&>-10 \end{aligned}$$

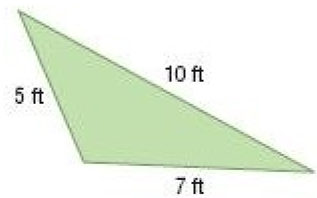
Therefore, the inequality is defined for $c>-5$.

The domain of the function $f(c)=\sqrt{c+5}+\sqrt{c+10}$ is $D=\{c\,|\,c>-5\}$.

The range of the function $f(c)$ is $\mathrm{R}=\left\{f(c)\,\middle|\,f(c)>\sqrt{5}\right\}$.

Since the lower limit of the range of the function itself is $\sqrt{5}>2$, the solution of the inequality $\sqrt{c+5}+\sqrt{c+10}>2$ is $c>-5$.

40. **GEOMETRY** The area of a triangle with sides of length a, b, and c is given by $A=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$. What is the area of the triangle expressed in radical form?



SOLUTION:

Substitute 5, 7 and 10 for a, b and c.

$$\begin{aligned} s &= \frac{1}{2}(5+7+10) \\ &= \frac{22}{2} \\ &= 11 \end{aligned}$$

$$\begin{aligned} A &= \sqrt{11(11-5)(11-7)(11-10)} \\ &= \sqrt{11(6)(4)(1)} \\ &= 2\sqrt{66} \end{aligned}$$

The area of the triangle is $2\sqrt{66}$ ft^2.